Supplementary Information

Database: In the present study we used a photometric database collected over the last 20 years for 21 Galactic globular clusters (GCs). In each cluster the central regions have been typically observed in the ultraviolet band with the Wide Field Planetary Camera 2 (WFPC2) on board the Hubble Space Telescope (HST, under programs GO-11975, GO-10524, GO-8709, GO-6607, GO-5903), possibly combined with complementary optical observations secured with the HST-Advanced Camera for Surveys. External regions are sampled by ground-based wide-field observations[10-13,17,20,24]. In all programme clusters the observations sampled a significant fraction (ranging from 70 to 100%) of the total cluster light.

The radial distribution of the reference population: In order to quantitatively study the BSS radial distribution, it is necessary to define a reference population. We chose to adopt two populations tracing the radial distribution of the parent cluster integrated light: the red giant and/or the horizontal branch stars. Their selection has been performed from the same photometric catalogues used for the BSS, in order to avoid any bias in the comparison.

Their observed distributions are shown in Figure 1 and Supplementary Figure 1 for a selection of GCs in different dynamical-age *Families* and are discussed in specific papers describing each individual cluster[10-13,17,20,24].

BSS from different formation channel: BSS are suggested to form through mass transfer/merger in binary systems (MT-BSS) and through stellar collisions (COL-BSS). These latter are thought to be generated essentially in the cluster cores[14] (and probably only in high collision-rate systems), while MT-BSS are more probable in the outskirts, where stellar densities are relatively low (this also confirmed by recent results obtained in open clusters[15]). Moreover, previous work[12,13] demonstrated that COL-BSS kicked out from the core sink back into the centre in a very short timescale (≤1 Gyr). Hence COL-BSS are expected to mainly/only contribute to the central peak of the observed BSS radial distribution. Instead, the portion of the distribution beyond the cluster core (and thus the definition of r_{min}), is essentially due to MT-BSS and has been shaped by the effect of dynamical friction for a significant fraction of the cluster lifetime. In fact, the progenitors of MT-BSS (i.e., primordial binaries of ~1.2 $M_{\odot}$) are the most massive objects in a cluster since ~7 Gyr (the main sequence lifetime of a 1.3 $M_{\odot}$ star being ~5 Gyr) and, given the shape of any reasonable stellar initial mass function, they are expected to be more massive than the average since the very beginning of the cluster history (~12 Gyr).

Operative definition of r_{min}: The adopted value of r_{min} in each programme cluster corresponds to the centre of the radial bin of the BSS radial distribution where the lowest value of the double normalized ratio (R_{BSS}) is observed. Only in a few cases where two adjacent radial bins showed approximately the same values of R_{BSS}, the average of the two radii has been adopted.

N-body simulations: Our simulations are based on the direct summation code NBODY6 [30,31],which employs regularization techniques guaranteeing an exact treatment of interactions between stars, without the need of softening. Three populations of stars with different masses are simulated: the heavy Blue Straggler Stars (BSS), the intermediate-mass Red Giant Branch (RGB) stars and the lightest class Main Sequence (MS) stars. The ratios between the masses assigned to each class are 3:2:1, that is RGB stars are twice as heavy as MS stars and BSS are three times as heavy as MS stars. This is consistent with a typical assumption of an average mass of 0.4 $M_{\odot}$ for MS stars, 0.8 $M_{\odot}$ for RGB and 1.2 $M_{\odot}$ for BSS. The number of objects in each class is such that MS stars are 89% of the total, RGB stars are 10% and BSS are the 1%. As the cluster evolves and thereby loses mass, the ratio between the populations changes slightly. From the astrophysical point of view, the amount of BSS is far in excess of what is typically observed in GCs, but this choice is dictated by the need of having a number of BSS suitable for statistical purposes, notwithstanding the low number of particles that we are able to simulate on present-day computers. On the other hand, such an assumption is expected to have no impact on the overall dynamical evolution of the system.
The initial conditions of the simulations are set as King models[28] with W0 = 6, corresponding to a relatively low concentration parameter, c = 1.25. The masses of stars are assigned at random (but respecting the above proportions), ensuring the presence of no mass segregation in the initial conditions (in agreement with Figure 1). No primordial binaries are included in the simulations. We performed 8 simulations of 16k particles each, with the same initial concentration, mass and number ratios. About six thousand snapshots were extracted from each simulation, allowing a fine-grained observation of the dynamical evolution of the clusters. The snapshots consist of tables containing the position, velocities and masses of all the stars still present in the system at a given time. We analyzed each snapshot as follows:

(1) We projected the position of each star onto three orthogonal planes, thus obtaining three times more stars

(2) We counted the number of BSS and RGB stars in concentric radial bins

(3) We calculated R_{BSS}, normalized to the RGB population, in each bin and its error based on Poisson counting statistics

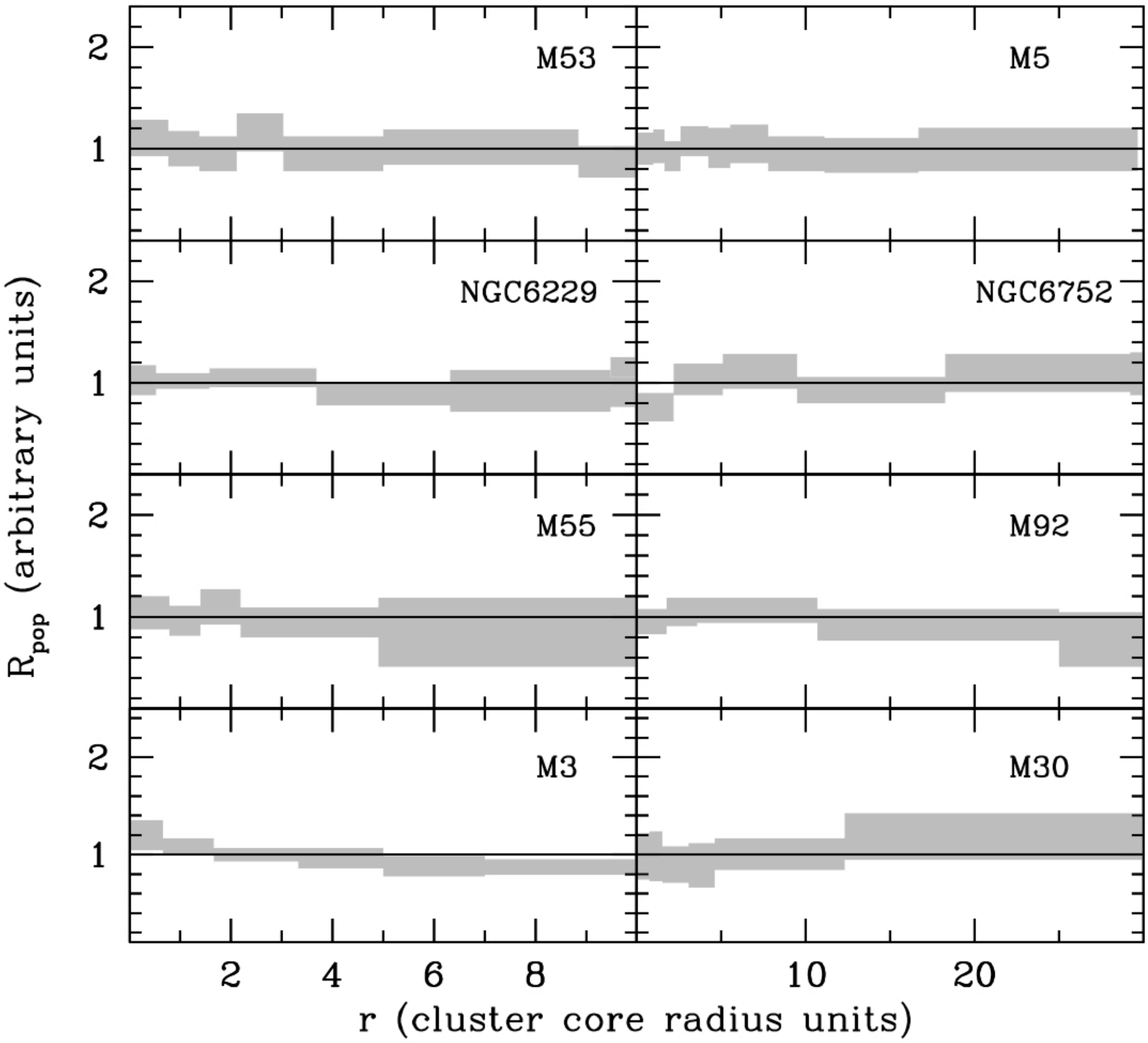
(4) We determined the position of the minimum of the radial distribution (if any).

The results are shown in Supplementary Figure 2, that confirms the progressive outward migration of the minimum of the BSS distribution as a function of time. While the purpose of these simulations is just to capture and illustrate the fundamental behaviour of the segregation process and the time evolution of the proposed dynamical age indicator ($r_{\rm min}$), we emphasize that for a detailed comparison with observations larger and more realistic simulations and initial conditions are needed.

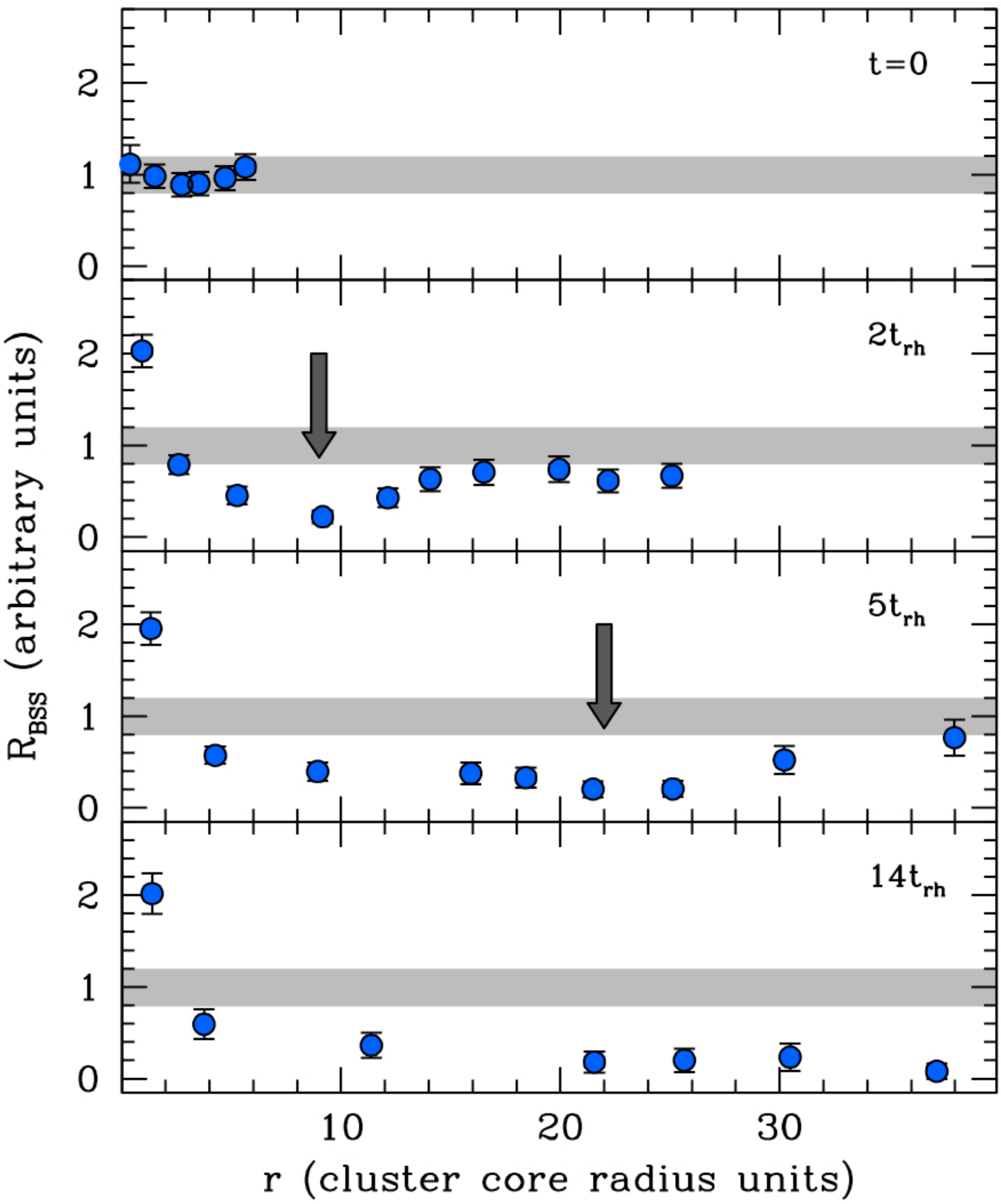
Core collapse and BSS in highly evolved clusters: The core collapse is a catastrophic dynamical process consisting in the runaway contraction of the core of a star cluster. About 15% of the GC population in our Galaxy shows evidence of a steep central cusp in the projected star density profile, a feature commonly interpreted as the signature of core collapse[1,27]. The BSS distribution provides precious information about this extreme stage of cluster evolution. In fact, binary-burning activity has been suggested to halt (or delay) the collapse of the core and it could be the origin of the large and highly centrally segregated population of BSS observed in M80[32], while the recent discovery of two distinct sequences of BSS in the post core collapse cluster M30 has opened the possibility of quantitatively dating the core collapse event[20].

Relaxation time. The relaxation time of a stellar system is defined as the characteristic time-scale over which stars lose memory of their orbital initial conditions. It is commonly calculated[1] using either the cluster central properties (central relaxation time $t_{\rm rc}$) or those within the radius enclosing half of the cluster total mass (half-mass relaxation time $t_{\rm rh}$).

Supplementary Figures



Supplementary Figure 1. The radial distribution of the reference population. A selection of the observed radial distributions of red giant/horizontal branch stars in a representative sample of clusters belonging to different dynamical-age *Families* is shown. The radial distribution of the double normalized ratio (R_{pop}) is always centred around unity, as expected for any population for which the number density scales with the luminosity sampled in each radial bin[10,25].



Supplementary Figure 2. BSS radial distributions obtained from direct N-body simulations. The double normalized BSS ratio, computed with respect to the red giant population ($R_{BSS}(r)$= $[N_{BSS}(r)/N_{BSS,tot}]$ / $[N_{RGB}(r)/N_{RGB,tot}]$) is shown for four snapshots at increasing evolutionary times (see labels), suitably selected to highlight the outward drift of r_{min}. The grey band around unity is drawn just for reference. By construction (in agreement with Figure 1) the initial conditions are mass-segregation free and BSS are distributed in the same way as red giant stars in every bin (top panel). After a couple of relaxation times a minimum forms in the BSS distribution and then its position progressively moves outward with time, in agreement with the observational results shown in Figure 2. At the latest stage of the evolution (bottom panel), the BSS distribution is centrally peaked and monotonically declining with radius (as in Figure 3). These results show that there is a clear connection between $r_{\rm min}$ and time, fully confirming our interpretation of $r_{\rm min}$ as time-hand of the dynamical clock. However, a quite large scatter is found at several (>13) relaxation times, probably due to counting noise and the progressive disappearing of the rising branch that do not allow us to clearly identify the position of the minimum.

Supplementary Table

Table S1. Parameters for the clusters in the sample.

Name	**c**	r_c	r_{min}	**log(**t_{rc}**)**	**log(**t_{rh}**)**	**Dynamical-age Family**
ω Centauri	1.31	153	--	9.86	10.59	*I*
NGC 2419	1.36	20	--	10.08	10.88	*I*
Palomar 14	0.88	41	--	9.68	10.09	*I*
M53	1.58	26	55	9.08	10.10	*II*
NGC 288	0.98	88	250	9.19	9.64	*II*
M55	1.01	114	405	9.15	9.63	*II*
NGC 6388	1.82	7.2	32.5	8.08	9.59	*II*
M4	1.60	70	350	8.00	9.19	*II*
NGC 6229	1.49	9.5	25	8.72	9.58	*II*
M3	1.77	30	125	8.75	10.17	*II*
M13	1.48	34	185	8.61	9.69	*II*
M2	1.51	17	150	8.48	9.52	*II*
M10	1.38	48	425	8.44	9.27	*II*
M5	1.68	27	255	8.43	9.69	*II*
47 Tucanae	1.95	21	200	7.96	9.81	*II*
M92	1.76	14	250	8.05	9.49	*II*
NGC 6752	2.09	13.7	325	7.37	9.47	*II*
M75	1.75	5.4	225	8.00	9.38	*III*
M79	1.71	9.7	325	7.98	9.30	*III*
M80	1.74		375	7.57	9.04	*III*
M30	2.29	4.3	385	6.79	9.51	*III*

Concentration (c), core radius in arcseconds (r_c), position of the BSS distribution minimum in arcsecond (r_{min}), central and half-mass relaxation times in years (t_{rc} and t_{rh}, respectively), dynamical-age family.

Supplementary References

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