Using long-term millisecond pulsar timing to obtain physical characteristics of the bulge globular cluster Terzan 5.

Brian Prager
Department of Astronomy
University of Virginia

Collaborators: Scott Ransom (NRAO), Phil Arras (UVa), Paulo Freire (NAIC), Jason Hessels (ASTRON), Ingrid Stairs (University British Columbia), Ryan Lynch (NRAO)
Terzan 5 is a globular cluster in the bulge of the Milky Way.

Among its unique properties are:

- 34 confirmed millisecond pulsars (MSPs). This represents ~25% of all known pulsars in GCs. (Ransom et al. 2005; Hessels et al. 2006)
- One of the largest interaction rates for any globular cluster. (Verbunt et al. 1987)
- Three separate stellar populations (Ferraro et al. 2009; Origlia et al. 2013), and may be a stripped dwarf galaxy. (Massari et al. 2014)
Terzan 5 is a difficult cluster to study for a number of reasons as well:

- Average $E(B-V)$ of 2.35 (Barbuy et al. 1998; Valenti et al 2007)
- Strong differential reddening. (Massari et al. 2012)
- Severe stellar crowding.
Motivations

Terzan 5 is a difficult cluster to study for a number of reasons as well:

- Average E(B-V) of 2.35 (Barbuy et al. 1998; Valenti et al 2007)
- Strong differential reddening. (Massari et al. 2012)
- Severe stellar crowding.

Traditional observations require either space based telescopes or good adaptive optics to overcome these issues.
Alternative Observational Methods

- Pulsar timing lets us measure cluster induced accelerations that do not suffer from strong reddening or stellar crowding.
- These accelerations are entirely due to gravitational effects, which gives the mass density profile of the system.
Pulsar timing as an acceleration

• Changes in the timing properties of a pulsar can be related to the acceleration along our line of sight.
• Decomposing a measured period change (spin or orbital) into its component parts yields:

\[
\begin{align*}
\left( \frac{\dot{P}}{P} \right)_{\text{meas}} &= \frac{a_c}{c} + \left( \frac{\dot{P}}{P} \right)_{\text{int}} + \frac{a_g}{c} + \frac{a_s}{c} + \frac{a_{\delta_{DM}}}{c}
\end{align*}
\]
Pulsar timing as an acceleration

- Changes in the timing properties of a pulsar can be related to the acceleration along our line of sight.
- Decomposing a measured period change (spin or orbital) into its component parts yields:

\[
\begin{align*}
\left( \frac{\dot{P}}{P} \right)_{\text{meas}} &= \frac{a_c}{c} + \left( \frac{\dot{P}}{P} \right)_{\text{int}} + \frac{a_g}{c} + \frac{a_s}{c} + \frac{a_{\delta_{DM}}}{c}
\end{align*}
\]

Acceleration due to the cluster potential.
Pulsar timing as an acceleration

- Changes in the timing properties of a pulsar can be related to the acceleration along our line of sight.
- Decomposing a measured period change (spin or orbital) into its component parts yields:

\[
\left( \frac{\dot{P}}{P} \right)_{\text{meas}} = \frac{a_c}{c} + \left( \frac{\dot{P}}{P} \right)_{\text{int}} + \frac{a_g}{c} + \frac{a_s}{c} + \frac{a_{\delta_{DM}}}{c}
\]

Period changes that are intrinsic to the system.
Pulsar timing as an acceleration

- Changes in the timing properties of a pulsar can be related to the acceleration along our line of sight.
- Decomposing a measured period change (spin or orbital) into its component parts yields:

\[
\left( \frac{\dot{P}}{P} \right)_{\text{meas}} = \frac{\alpha_c}{c} + \left( \frac{\dot{P}}{P} \right)_{\text{int}} + \frac{\alpha_g}{c} + \frac{\alpha_s}{c} + \frac{\alpha_{\delta_{DM}}}{c}
\]

Acceleration due to galactic rotation.
Pulsar timing as an acceleration

- Changes in the timing properties of a pulsar can be related to the acceleration along our line of sight.
- Decomposing a measured period change (spin or orbital) into its component parts yields:

\[
\left( \frac{\dot{P}}{P} \right)_{\text{meas}} = \frac{a_c}{c} + \left( \frac{\dot{P}}{P} \right)_{\text{int}} + \frac{a_g}{c} + \frac{a_s}{c} + \frac{a_{\delta_{DM}}}{c}
\]

Acceleration due to the proper motion of the source.
Pulsar timing as an acceleration

- Changes in the timing properties of a pulsar can be related to the acceleration along our line of sight.
- Decomposing a measured period change (spin or orbital) into its component parts yields:

\[
\left( \frac{\dot{P}}{P} \right)_{\text{meas}} = \frac{a_c}{c} + \left( \frac{\dot{P}}{P} \right)_{\text{int}} + \frac{\alpha_g}{c} + \frac{\alpha_s}{c} + \frac{a_{\delta DM}}{c}
\]

Accelerations due to unaccounted changes in the dispersion measure (DM) can affect spin period measurements.
From Reid et al. (2014) we can solve for the galactic acceleration on each pulsar.

The proper motion and DM error of each pulsar is small enough to be negligible (~0.01\% of the measured accelerations).

This leaves intrinsic cluster accelerations to consider.
Modeling Terzan 5

- Pulsar studies of globular clusters have been carried out in the past for clusters such as M15 and 47 Tucanae. (Phinney 1993; Anderson 1993; Freire et al. 2001B, 2003)

- These studies make use of the single most likely line of sight position of the pulsars to obtain their results.

- Using a Markov Chain Monte Carlo (MCMC) method, we assign a 3-d position for each pulsar and compare the predicted acceleration measurements to our model.
We model the line of sight acceleration using an integrated King profile (King 1962). The general form of the acceleration profile is given by:

\[ a_l(l, r) = 1.17 \times 10^{-7} \left( \frac{\rho_c}{10^6 M_\odot pc^{-3}} \right) \left( \frac{l}{2 pc} \right) \times _2F_1 \left( \frac{3}{2}, \frac{\alpha - 1}{2}; \frac{5}{2}; - \left( \frac{r}{r_c} \right)^2 \right) \text{ m s}^{-2} \]

where \( \rho_c \) is the core density, \( \alpha \) is related to the mass ratio of neutron stars to main sequence turn-off stars, \( r_c \) is the core radius, and \( l \) is the line of sight position of the pulsar.
Model Acceleration

- We model the line of sight acceleration using an integrated King profile (King 1962).
- The general form of the acceleration profile is given by:

\[
\alpha_l(l, r) = 1.17 \times 10^{-7} \left( \frac{\rho_c}{10^6 M_\odot pc^{-3}} \right) \left( \frac{l}{2 pc} \right) \times _2F_1 \left( \frac{3}{2}, \frac{\alpha}{2}; \frac{5}{2}; - \left( \frac{r}{r_c} \right)^2 \right) \text{m s}^{-2}
\]

where \( \rho_c \) is the core density, \( \alpha \) is related to the mass ratio of neutron stars to main sequence turn-off stars, \( r_c \) is the core radius, and \( l \) is the line of sight position of the pulsar.
Model Acceleration

- We simulated the mean-field and nearest neighbor accelerations using the C based artificial star cluster initializer, McLuster (Kupper et al. 2011)
- Simulated cluster properties were taken from the findings of Miocchi et al. (2013)
- We find that the probability of the nearest neighbor being a significant acceleration in our timing is small, as predicted by Phinney (1993).
We model the characteristic jerk of the cluster as a function of radius

\[ \dot{a}_0 = \frac{2\pi \xi}{3} G < m\sigma > n \]

where \( \xi \) is a numerical prefactor, \( m \) is the average particle mass in a spherical shell, \( \sigma \) is the velocity dispersion, and \( n \) is the number density.

The nearest neighbors contribute as much to the jerk as the mean-field, as predicted by Blandford (1987).
Likelihood Functions - Accelerations

- Residuals of our model and the data have a distribution that is set by the pulsar losing rotational energy.
- The magnetic field strength of pulsars can be related to the change in pulsar spin rates.
- The magnetic field strength of pulsars follow a log-normal distribution, as observed in galactic field pulsars reported in the ATNF catalog. (Manchester et al. 2005)
Likelihood Functions - Jerks

- Assuming a Maxwell-Boltzmann velocity distribution and core radius much smaller than the tidal radius, we find a Lorentzian PDF for pulsar jerks.

Measured jerks from a simulated Terzan 5 cluster fall along a Lorentzian PDF defined at one core radii of the cluster.
IMBH Predictions

- If we know the radius of influence of the black hole (Baumgardt 2004a) we can see if any pulsars might be close enough to an IMBH that their accelerations are dominated by the BH and not the mean-field.
- If we know the density profile in the radius of influence (Baumgardt 2004b) we can modify the model acceleration profile to allow for the presence of black holes in our simulation.

\[
r_i = \frac{3M_{BH}}{8\pi \rho_c r_c^2}
\]

\[
\rho_{BH} \propto r^{1.55}
\]

\[
a_i(l, r) = \frac{4\pi G \, l}{r^2} \frac{1}{r} \left[ \int_0^{r_i} r^2 \rho_{BH} dr + \int_{r_i}^{r} r^2 \rho_{\text{King}} \right]
\]
Cluster Parameter Fits

Acceleration Fitting Only

\[ \rho_c = (0.92 \pm 0.12) \times 10^6 \, M_\odot \, pc^{-3} \]
\[ r_c = 0.17 \pm 0.02 \, pc \]
\[ \alpha = 3.45 \pm 0.21 \]

Acceleration + Jerk Fitting

\[ \rho_c = (0.89 \pm 0.14) \times 10^6 \, M_\odot \, pc^{-3} \]
\[ r_c = 0.18 \pm 0.03 \, pc \]
\[ \alpha = 3.48 \pm 0.42 \]
No Blackhole in Terzan 5 greater than \(\sim 1300\ M_\odot\).
Preliminary results for 47 Tucanae.

- We have also applied this fitting routine to 47 Tucanae.
- We find that the fits are not very constraining without more pulsars near the center of gravity.
- Our results agree with those found by Freire et al. 2001 and Giersz et al. 2011 to within our error-bars.
Questions?

<table>
<thead>
<tr>
<th></th>
<th>Optical/IR</th>
<th>Pulsar Timing</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_c )</td>
<td>( 1 \times 10^6 , M_\odot/pc^3 )(^1)</td>
<td>( 0.9 \times 10^6 , M_\odot/pc^3 )</td>
</tr>
<tr>
<td>( r_c )</td>
<td>( 0.26 , pc )(^1) ( 0.22 , pc )(^2)</td>
<td>( 0.17 ) (Neutron Stars) ( 0.20 ) (MSTO Equivalent)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>( 3.3 )(^3)</td>
<td>( 3.5 )</td>
</tr>
<tr>
<td>( M_{\text{total}} )</td>
<td>( 2 \times 10^6 , M_\odot )</td>
<td>( 0.75 \times 10^6 , M_\odot )(^4)</td>
</tr>
<tr>
<td>( M_{\text{BH}} )</td>
<td></td>
<td>( \leq 1300 , M_\odot )</td>
</tr>
</tbody>
</table>

1) Lanzoni et al. 2010  
2) Miocchi et al. 2013  
3) Heinke et al. 2006  
4) Found by integrating only the single mass King model for our neutron star results.
Spare Slides – Line of Sight Accelerations
## Table 4

Results of Parallel Tempered MCMC Analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial Fit</th>
<th>Accelerations</th>
<th>Jacobian</th>
<th>Acceleration &amp; Jerk</th>
<th>Secondary</th>
<th>Jacobian</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_c )</td>
<td>0.94 ( \times 10^6 ) M(_{\odot}) pc(^{-3})</td>
<td>0.94 ( \times 10^6 ) M(_{\odot}) pc(^{-3})</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>( v_c )</td>
<td>0.18 ( \times 10^6 ) pc</td>
<td>0.18 ( \times 10^6 ) pc</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>3.5 \pm 0.24</td>
<td>4.25 \pm 0.25</td>
<td>N/A</td>
<td>N/A</td>
<td>4.12 \pm 0.54</td>
<td>N/A</td>
</tr>
<tr>
<td>( M_{\text{tot}} )</td>
<td>3.8</td>
<td>1.7</td>
<td>N/A</td>
<td>N/A</td>
<td>1.9</td>
<td>N/A</td>
</tr>
<tr>
<td>( t_1 )</td>
<td>N/A</td>
<td>N/A</td>
<td>1.29 \pm 0.19</td>
<td>11.18</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>( t_2 )</td>
<td>N/A</td>
<td>N/A</td>
<td>1.000</td>
<td>0.000</td>
<td>1.44 \pm 0.19</td>
<td>8.203</td>
</tr>
<tr>
<td>( t_3 )</td>
<td>N/A</td>
<td>N/A</td>
<td>1.000</td>
<td>0.000</td>
<td>0.179 \pm 0.037</td>
<td>0.690</td>
</tr>
<tr>
<td>( t_4 )</td>
<td>N/A</td>
<td>N/A</td>
<td>0.000</td>
<td>0.000</td>
<td>1.360 \pm 0.069</td>
<td>1.403</td>
</tr>
<tr>
<td>( t_5 )</td>
<td>N/A</td>
<td>N/A</td>
<td>1.000</td>
<td>0.000</td>
<td>0.199 \pm 0.033</td>
<td>0.000</td>
</tr>
<tr>
<td>( t_6 )</td>
<td>N/A</td>
<td>N/A</td>
<td>0.000</td>
<td>0.000</td>
<td>0.208 \pm 0.032</td>
<td>0.000</td>
</tr>
<tr>
<td>( t_7 )</td>
<td>N/A</td>
<td>N/A</td>
<td>1.000</td>
<td>0.000</td>
<td>0.415 \pm 0.129</td>
<td>0.415</td>
</tr>
<tr>
<td>( t_8 )</td>
<td>N/A</td>
<td>N/A</td>
<td>0.000</td>
<td>0.000</td>
<td>0.404 \pm 0.155</td>
<td>0.404</td>
</tr>
<tr>
<td>( t_9 )</td>
<td>N/A</td>
<td>N/A</td>
<td>0.000</td>
<td>0.000</td>
<td>0.357 \pm 0.158</td>
<td>0.357</td>
</tr>
<tr>
<td>( t_{10} )</td>
<td>N/A</td>
<td>N/A</td>
<td>0.000</td>
<td>0.000</td>
<td>0.390 \pm 0.161</td>
<td>0.390</td>
</tr>
<tr>
<td>( t_{11} )</td>
<td>N/A</td>
<td>N/A</td>
<td>1.000</td>
<td>0.000</td>
<td>1.430 \pm 0.319</td>
<td>7.836</td>
</tr>
</tbody>
</table>

Note: Final fits to cluster parameters using MCMC analysis of pulsar timing. Included are the core density, core radius, spectral index of pulsar number density, and the line of sight position of each source. We include the line of sight position for each pulsar for both the primary solution and the secondary solution, as well as the relative probability between the two. We also introduce results using just acceleration fits as well as acceleration plus jerk fitting.

1. Secondary solution was of such small probability after using the jerks that it was not measurable above the tail of the primary distribution.
Spare Slides – Projected Accelerations

\[ a \left[ 10^{-9} \text{ m s}^{-2} \right] \]

\[ R_{\perp} / r_c \]

- \( \rho_c = 0.55^{+0.13}_{-0.09} \times 10^6 \text{ M}_\odot \text{ pc}^{-3} \)
- \( r_c = 0.17^{+0.07}_{-0.03} \text{ pc} \)
- \( v_c = 17.32^{+2.87}_{-2.18} \text{ km/s} \)
We find the observed distribution of pulsar velocities agrees with the expected distribution.
Spare Slides - Animation